Calc BC - 10.2 Vectors

2016 * [Gave s's handout & problem sets from College Board and had them finish them over 2/3 days (says Days 4-6 on Packet)]

"Day 4" homework

① $x = t^2 - 1$, $y = e^{t^3}$ $\quad \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3t^2 \cdot e^{t^3}}{2t} = \dfrac{3t \cdot e^{t^3}}{2}$

② Position: $\langle \ln(t^2+5t), 3t^2 \rangle$ $\quad v(t) = \left\langle \dfrac{2t+5}{t^2+5t}, 6t \right\rangle$

$v(t)\big|_{t=2} = \left\langle \dfrac{9}{14}, 12 \right\rangle$

③ $v(t) = \left\langle \dfrac{dx}{dt}, \dfrac{dy}{dt} \right\rangle = \langle 5t^4, 12t^3 - 6t^2 \rangle$

$a(t) = \langle 20t^3, 36t^2 - 12t \rangle$, $\quad a(t)\big|_{t=1} = \langle 20, 24 \rangle$

④ $v(t) = \langle 3\cos(3t - \frac{\pi}{2}), 6t \rangle$, $\quad v(t)\big|_{t=\frac{\pi}{2}} = \langle -3, 3\pi \rangle$

⑤ $x'(t) = t + 1$

Position $= 1 + \int_0^1 x'(t)\,dt$

$= 1 + \int_0^1 t+1\,dt = 1 + \left[\frac{1}{2}t^2 + t\right]_0^1 = 2\frac{1}{2}$

$y = \ln x \Rightarrow y(1) = \ln(\frac{5}{2})$ ($\frac{5}{2}$ is the x-value @ $t=1$)

Position $= \left(\frac{5}{2}, \ln(\frac{5}{2})\right)$

⑥ $v(t) = \langle 1+t, t^3 \rangle$

$x(2) = 5 + \int_0^2 1+t \, dt = 5 + \left[t + \frac{1}{2}t^2 \right]_0^2 = 5 + 2 + 2 = 9$

$y(2) = 0 + \int_0^2 t^3 \, dt = \frac{1}{4}t^4 \Big]_0^2 = 4$

Position : $(9,4)$

⑦ $xy = 10$ when $x=2$, $y=5$

$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0 \Rightarrow \frac{dx}{dt} \cdot 5 + 2(3) = 0 \Rightarrow \frac{dx}{dt} = -\frac{6}{5}$

⑧ $v(t) = 0$, $\frac{dx}{dt} = 3t^2 - 3t - 18 = 3(t^2 - t - 6) = 0$

$3(t-3)(t+2) = 0$ when $t = 3$
$t = -2$

$\frac{dy}{dt} = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$ when $t = 3$
$t = 1$

$v(t) = \langle 0, 0 \rangle$ at $t = 3$

(9) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-5}{3t^2}$ Need a t. $t^3 = 8$, $t = 2$ | $t^2 - 5t + 2 = -4$, $t^2 - 5t + 6 = 0$, $(t-3)(t-2) = 0$, $t = 3,\ t = 2$

use $t = 2$, $\left.\frac{dy}{dx}\right|_{t=2} = \frac{-1}{12}$

$$y + 4 = -\frac{1}{12}(x - 8)$$

(10) $\frac{dx}{dt} = 5 + 3\cos t$, $\frac{dy}{dt} = (8-t)(\sin t) - (1 - \cos t)$

Set $5t + 3\sin t = 25$

$t = 5.446$ (from calc)

$$v(5.446) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle \Big|_{t=5.446} = \langle 7.008, -2.228 \rangle$$

(11) a) Magnitude $= \left.\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\right|_{t=5}$

$$= \left.\sqrt{(2t)^2 + (2t^2)^2}\right|_{t=5}$$

$$= \left.\sqrt{4t^2 + 4t^4}\right|_{t=5}$$

$$= \sqrt{100 + 2500}$$

$$= \sqrt{2600} \text{ or } 10\sqrt{26}$$